In view of this or similar examples,[1] it is clear that if Duhamel's Theorem is to be used at all, as a means of securing rigor, it must be taken in a modified form; interesting revised forms have in fact been proposed by Professors Osgood, R. L. Moore, and Bliss (*loc. cit.*); but, while these new forms leave nothing to be desired in point of rigor, none of them, as far as I know, has proved to be sufficiently simple to warrant its adoption in an elementary textbook.[2] The simplest plan to pursue in a first course in the calculus would therefore appear to be to omit Duhamel's Theorem altogether, substituting for it some such theorem as that suggested in the present paper.

BOOK REVIEWS.

SEND ALL COMMUNICATIONS TO W. H. BUSSEY, University of Minnesota.

Ruler and Compasses. By HILDA P. HUDSON. Longmans, Green and Company, London and New York, 1916. 143 pages.

This new volume of Longmans' Modern Mathematical Series is an attempt to collect from many sources solutions of problems and discussions of methods in which the Euclidean ruler and compasses are used as instruments; and to present them as part of a well-ordered development of the theory of such constructions. According to the author "the connecting link throughout the book is the idea of the whole set of ruler and compass constructions, its extent, its limitations, and its division."

The reader will require no more advanced mathematics than college algebra and elementary analytic geometry, although a knowledge of projective geometry and of the theory of equations in general will be helpful. The development of the theme is carefully carried out and there are no breaks in the logic, although in one or two places the author quotes a theorem which is developed later. This may prove a bit annoying to the reader with a minimum of preparation, but otherwise it is not a serious fault.

The subject matter of the text is presented as a whole in the introduction, which is rather well written, although it presupposes at times a rather full acquaintance with the material which is developed later. In Chapter II the criteria of possibility for ruler and compass constructions are established from the analytical point of view with the aid of a number of propositions from the elementary theory of equations. The chapter is divided into three parts; first, the constructions in which the ruler alone suffices, second, those in which the ruler and Euclidean compasses are required, and third, the construction of regular polygons of n sides. The cases in which n is a prime and n is composite

[1] During the discussion of this paper at the meeting of the Society, Professor D. Jackson suggested the following even simpler example: $\alpha_i = 1/n$ when $i \leqq n/2$, $\alpha_i = 0$ when $i > n/2$; $\beta_i = 1/n$.

[2] Even in Professor Osgood's own text (1907, revised edition 1909), the original (incorrect) form of Duhamel's theorem is retained, without comment. Professor Osgood's reasons for so doing may be found in his article of 1903 (*loc. cit.*).

are discussed in the classical way and Richmond's construction for the polygon of 17 sides is given as an example. In the third chapter the question of constructions to be carried out by ruler alone is developed in detail with the separation of projective and metrical properties. Mathew's *Projective Geometry* is drawn upon extensively for the discussion of homography and its special case of involution to establish the constructions. The relation of the infinity locus to metrical constructions is unusually well presented.

In Chapter IV the fundamental theorem concerning the possibility of solution of a quadratic equation with the aid of ruler and compasses is established, two solutions being given, and, from the discussion of a pencil of conics projected from a circle through four points, a construction for the common points of projective point rows superposed on a line is obtained. A second division of this chapter treats of modern instruments, including dividers, Hilbert's Einheitsdreher, the parallel ruler, and the set-square. In Chapter V standard methods of attack for various problems are discussed. No pretense is made at developing a general method for all problems, but the data in any given case are used to point toward one of seven listed methods of solution. This is in some respects the best organized chapter of the book. In Chapter VI the methods just indicated are compared, especially as regards constructions to be accomplished in a limited space or with a minimum number of operations. The two concluding chapters are concerned with effecting all ruler and compass constructions with one fixed circle and ruler only, or with compasses only. They may be regarded as addenda to the preceding discussion, inasmuch as they are not necessary to its development. In the chapter on compasses only, the results of Mascheroni and Adler have been compared and several solutions by the methods of each given as examples.

On the whole, the teacher of analytic or projective geometry or of college algebra will find this a valuable source-book for many illustrative problems. Its development of fundamental theorems of projective geometry for the general conic starting from properties of the circle is carried out in an interesting way. The sources of many theorems are carefully indicated and a fairly complete bibliography is given facing page 1. In the way of criticism, the reviewer believes that some of the section headings are misplaced and that many of them are not illuminating. The chapter on methods of solution is an exception to this criticism.

B. M. Woods.

The University of California.

A Treatise on the Circle and the Sphere. By Julian Lowell Coolidge, Assistant Professor of Mathematics in Harvard University. The Clarendon Press, Oxford, 1916. 604 pages. $6.75.

Professor Coolidge has written a treatise on the circle and the sphere which may be considered as an encyclopedia of valuable information on this important subject. The successful compilation of known material of such an extent, and the incorporation of the results of his own notable investigations in this field,